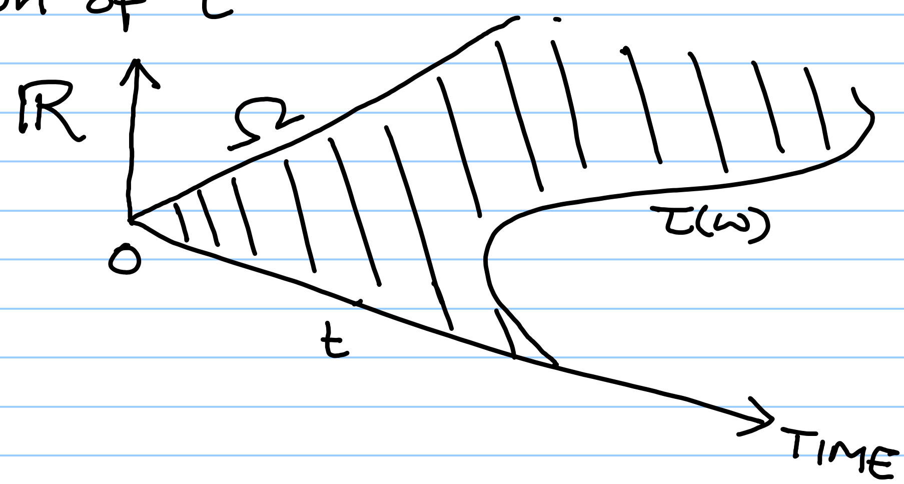
Local Objects

Our context is a filtration satisfying the usual conditions. Inside the set of all processes adapted to this filtration there are many classes of processes; bounded variation, martingales, supermartingales, submartingales, semi-martingales... etc. All of these are defined by properties that occur at each time, $t \in [0, \infty]$. A process, (X_t), can share the property of being, say, a martingale, if there is a stopping time, τ, say, such that $(X_{\tau \wedge t}) \equiv X^\tau$ is a martingale. But (X_t) need not be a martingale itself for this to occur. What we are saying is that if we restrict attention to the region of $[0, \infty] \times \Omega$ that lies between 0 and the graph of τ



then (X_t) behaves like a martingale here.

A process $X = (X_t)$ is a local martingale if there is a sequence of stopping times, $0 \leq \tau_1 \leq \tau_2 \leq$ ascending pointwise to $+\infty$ such that each of the processes $X^{\tau_n} \equiv (X_{\tau_n \wedge t})$ is a (uniformly integrable) martingale. You can replace 'martingale' with 'increasing process', 'supermartingale' and so on. The sequence of times (T_n) is said to be a fundamental seque-nce for X.

Not all local martingales are martinga-les. There are 'famous' examples and some more mundane ones arising from the Itô integral. See the exercises.